Induced Polarization Effects in Coupling Processes of Waveguide Modes

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Abstract— **An analysis of waveguide problems based on a solution of full vector-wave equations is very important for many applications. To solve such problems, a new coupled-mode method, taking into account the so-called induced polarization effects, is proposed in this paper. The theory is based on the spectral method, which makes it possible to analyze correctly a mode excitation by arbitrary sources with longitudinal components. It also takes into consideration singularities caused by abrupt discontinuities of longitudinal currents. The method may be a powerful tool for investigation of propagating and evanescent modes coupling due to both material and geometric effects.**

I. Introduction

A SOLUTION to the excitation problem in waveguides may be carried out with the so-called projective method, which employs the basis of normal modes of a regular waveguide as a system of trial and weight functions. Such a method allows us to attract the coupled-mode formalism—a very powerful tool in the analysis of various waveguide problems [1]–[3]. It has been shown in [4] and [5] for a wide class of waveguides (double anisotropic nonchiral, isotropic chiral) that in the presence of longitudinal currents as sources of excitation, it is more mathematically correct to use the so-called spectral method rather than the reciprocity relation in the form used in [1]. In this relation, we essentially apply a system of mode transverse fields as a system of trial functions and a system of mode full fields as a system of weight functions. On the other hand, one can obtain the excitation equation on the basis of a system of mode transverse fields as trial and weight functions. It enables us to apply correctly the Galerkin method for the solution of inhomogeneous Maxwell equations [4], [5].

The main inference, which one can conclude from the theories in [4] and [5], is that the efficiency of excitation is determined by both the values of transverse currents and the transverse derivatives of longitudinal currents. Therefore, the abrupt discontinuities of the longitudinal currents on the waveguide cross section cause the excitation by delta functions. However, as it will be shown below, one can obtain another approach without using delta functions. In this approach, we have two excitation integrals. The first is the integral over the cross section of excitation region S_j and the second is the integral over the contour surrounding S_j. Two such approaches are similar to those considered in [6] for the free-space scattering problem inside as well as outside the source region.

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In this paper we apply the theory [4], based on the spectral method, for development of the coupled-mode formalism by taking induced polarization effects into account. Such effects are caused by abrupt discontinuities of longitudinal polarization currents and have to be considered in a large variety of mode-coupling processes. We will concentrate on two problems: the coupled-mode theory of parallel dielectric waveguides and mode coupling in isotropic discretely inhomogeneous waveguides. These problems may be indicated in the following way.

A. Coupled-Mode Theory of Parallel Dielectric Waveguides

In a scope of the so-called improved coupled-mode theory of parallel dielectric waveguides, some ambiguities concerning the role of the axial electric field component take place. It gives different expressions in the theories presented in [7] and [8] for the coupling coefficients. The discussions in [9] and [10] about the question of what is the best approach (based on the reciprocity theorem [7] or the variational principle [8]) do not give us the comprehensive answer. It has been pointed out in [11] that the vector-formulated improved coupled-mode theory gives a fundamental error for strongly guiding structures in the case where fields are not pure TE waves. To disperse of such a fundamental error, the authors of [12] used the theory [8], taking into account the correction field caused by induced polarization charges. The problem, however, is not closed. Expressing the axial electric field according to [7] is more mathematically valid than using the field representation in [8] (this is shown in [1], [4], and [13]). The question, however, is about the correctness of using the reciprocity theorem for strongly guiding structures in the case that fields are not pure TE waves. The problem may be solved on the basis of the spectral method [4]. In this paper we will combine the theories of [7] and [4] for the analysis of strongly guiding structures.

B. Mode Coupling in Isotropic Discretely Inhomogeneous Waveguides

The transformation of TE and TM modes into hybrid modes in isotropic discretely inhomogeneous waveguides (IDIW) is shown in [14]–[17]. One of the interesting features of lossless IDIW is the presence of complex and backward-wave modes. Such modes can exist in a waveguide with a dielectric insert as modes of "hybrid-type" [15]–[18]. In a majority of papers devoted to the problem of complex and backward waves propagation, the spectral domain analysis is used. Nevertheless, in a number of papers, endeavors were made to explain the reasons that result in complex modes in the spectrum. In [19] and [20], complex waves in IDIW were predicted by the analysis of symmetry of a characteristic matrix.

In this paper, we consider the coupling of orthonormal (TE and TM) propagating and evanescent modes of a hollow metallic waveguide caused by insertion of a dielectric rod. Our analysis shows that by taking into account induced polarization effects, we obtain asymmetrical mode coupling in IDIW. According to our consideration, some conclusions stating that complex and backward waves in IDIW occur as a result of induced polarization effects in mode coupling may be made.

An investigation of mode coupling in IDIW may also be useful for other waveguide problems. A model of IDIW structure has been used recently for the analysis of optical waveguides [21]–[23]. It consists of enclosing the waveguide within a rectangle large enough to ensure that the fields of the guided modes of interest are zero at this boundary. In [21] and [22] the problem was solved on the basis of the scalar wave equation, but in [23] the analysis of full vector-wave equation was made. Optical waveguides, in comparison with millimeter or submillimeter waveguides, are usually weak guide structures. Therefore, the scalar solutions may be just as accurate as the vectorial solutions [24]. Nevertheless, one can see now a rising interest in optical waveguides with large refractive index differences concerning both theoretical aspects (mode solutions for single dielectric waveguide [23] or coupled-mode formulation for parallel dielectric waveguides [11], [12]) and applied problems for working out high-density integrated planar lightwave circuits [25]. If a waveguide has large refractive index differences, a vector solution is necessary. For such a solution one can use fully numerical finite-element or finite-difference methods [24]. Another approximate method based on the Galerkin method is shown in [23].

According to the Galerkin method, one expresses the vector field as series expansions in terms of a complete set of functions that otherwise may be quite arbitrary [26]. In [23] a complete orthogonal set of sine functions was used. As another type of complete orthogonal set, one can use the basic system (TE and TM modes) of a hollow metallic waveguide and consider the dielectric insert as some kind of irregularity. In such a case, the singularity caused by abrupt discontinuities of the permittivity on boundaries of dielectric insert is described by the induced polarization effects in the mode-coupling process. The problem formulated in this paper is similar to the vector-formulated problem in [23], but instead of two vector-wave equations (for electric and magnetic fields), we have only one vector equation.

The main goal of this paper is to demonstrate an appreciable role of induced polarization effects in mode-coupling processes for a wide class of waveguide problems. The paper may be conventionally divided into two main parts. The first part (Sections II, III, and the Mathematical Appendix) represents the general theory, and the second part (Sections IV, V, and VI) is devoted to some applications. Some numerical examples, which are adduced for a rectangular waveguide with a dielectric insert, enable estimating the importance of taking into account induced polarization effects in the mode coupling.

II. Excitation of Propagating and Evanescent Modes by Longitudinal Currents

Let us consider a regular waveguide of an arbitrary form of cross section S with the longitudinal z-axis. For any two modes, we have the orthogonality relation [4]

$$(\gamma_p + \tilde{\gamma}_q^*) \int_S (Q\hat{\vec{U}}_p)\hat{\tilde{\vec{U}}}_q^* \, ds = 0 \tag{1}$$

where

$$Q = \begin{pmatrix} 0 & -\vec{e}_z \times \\ \vec{e}_z \times & 0 \end{pmatrix} \tag{2}$$

$\vec{e}_z$ is the unit vector along z-axis, γ is the propagation constant

$$\hat{\vec{U}}_{p,q}(x,y) = \begin{pmatrix} \hat{\vec{E}}_{p,q}(x,y) \\ \hat{\vec{H}}_{p,q}(x,y) \end{pmatrix} \tag{3}$$

is the field function dependent on transverse coordinates of a waveguide.

For orthogonal modes we have the inequality $\gamma_p + \tilde{\gamma}_q^* \neq 0$. We will mark the mode with the number q, which satisfies the condition $\gamma_p + \tilde{\gamma}_q^* = 0$, as the mode with the number $\tilde{p}$. This mode is conjugate to the mode p. For conjugate modes we have an expression for the norm

$$\begin{aligned} N_p &= \int_S (Q\hat{\vec{U}}_{p\perp})\hat{\vec{U}}_{\tilde{p}\perp}^* \, ds \\ &= \int_S (\hat{\vec{E}}_{p\perp} \times \hat{\vec{H}}_{\tilde{p}\perp}^* + \hat{\vec{E}}_{\tilde{p}\perp}^* \times \hat{\vec{H}}_{p\perp}) \cdot \vec{e}_z \, ds \end{aligned} \tag{4}$$

where $\hat{\vec{E}}_\perp$ and $\hat{\vec{H}}_\perp$ are transverse components of the fields.

If modes are propagating ($\gamma = i\beta$), we have degeneration of conjugate modes: $\beta_p = \beta_{\tilde{p}}$. For evanescent modes ($\gamma = \alpha$), two modes with $\gamma_p = \alpha_p$ and $\gamma_{\tilde{p}} = -\alpha_p$ are conjugate modes. The norm (4) describes an active power flow through a waveguide cross section. Every propagating mode realizes a transfer of energy. In a case of evanescent modes, the carrying over of the energy may be realized only by pairs of modes. The modes of every pair are characterized by different signs of amplitude variation. In other words, the transmission of the energy by evanescent modes is possible only at a certain distance. For a single evanescent mode, the norm (4) is equal to zero. It is well known that in order to obtain a unitary generalized scattering matrix one has to have two cascaded junctions: above cutoff waveguide—below cutoff waveguide—above cutoff waveguide (see, for example, [27]). This means that the norm (4) is real quantity only if the below cutoff waveguide section will be loaded at the ends by two above cutoff waveguide sections. In such a case, the fields of modes p and $\tilde{p}$ are phase-shifted by the angle $\pi/2$.

Inhomogeneous Maxwell equations, in the region of excitation of an isotropic waveguide, we represent in the operator form

$$M\vec{U} = \vec{f} \tag{5}$$

where

$$M = \begin{pmatrix} i\omega\epsilon & -\nabla\times \\ \nabla\times & i\omega\mu \end{pmatrix} \tag{6}$$

is the Maxwell operator

$$\vec{U} = \begin{pmatrix} \vec{E} \\ \vec{H} \end{pmatrix} \tag{7}$$

$$\vec{f} = -\begin{pmatrix} \vec{j}^{el} \\ \vec{j}^{\,mag} \end{pmatrix} \tag{8}$$

$\vec{j}^{\,el}$ and $\vec{j}^{\,mag}$ are correspondingly electric and magnetic currents.

To solve (5), we express transverse components of the fields as a sum of transverse components of mode fields. In comparison with [4], we extend our analysis of excitation for both propagating and evanescent modes

$$\vec{U}_{\perp} = \sum_{m} a_m(z)\hat{\vec{U}}_{m\perp} + \sum_{p,\tilde{p}} (a_p(z)\hat{\vec{U}}_{p\perp} + a_{\tilde{p}}(z)\hat{\vec{U}}_{\tilde{p}\perp}) \tag{9}$$

where $a_m(z)$ is a scalar coefficient for propagating modes $(m = 1, 2, \cdots), a_p(z)$ and $a_{\tilde{p}}(z)$ are scalar coefficients for evanescent modes $(p = \tilde{p} = 1, 2, \cdots)$.

Analogous to the procedure worked out in [4], for the longitudinal components of the field in isotropic dielectric waveguide one can obtain

$$\vec{U}_{||} = \sum_{m} a_m(z)\hat{\vec{U}}_{m_{||}} + \sum_{p,\tilde{p}} (a_p(z)\hat{\vec{U}}_{p_{||}} + a_{\tilde{p}}(z)\hat{\vec{U}}_{\tilde{p}_{||}}) + \vec{U}_{ex} \tag{10}$$

where

$$\vec{U}_{ex} = \frac{i}{\omega}\begin{pmatrix} \frac{1}{\epsilon} & \vec{j}^{\,el}_{||} \\ \frac{1}{\mu} & \vec{j}^{\,mag}_{||} \end{pmatrix} \tag{11}$$

is the additional field in the excitation region caused by longitudinal parts of electric and magnetic currents.

We will consider below the relations for evanescent modes supposing that analogous relations for propagating modes may be easily obtained. One has the following evident relations

$$N_p \equiv \int_S (Q\hat{\vec{U}}_{p\perp})\hat{\vec{U}}^*_{\tilde{p}\perp}\, ds = \int_S \hat{\vec{U}}_{p\perp}(Q\hat{\vec{U}}_{\tilde{p}\perp})^*\, ds \equiv N^*_{\tilde{p}}. \tag{12}$$

Since the norm (4) (divided by 4) describes active power flow, we have $N^*_{\tilde{p}} = N_{\tilde{p}} = N_p$. The coefficients in (9) and (10) are determined on the basis of the orthogonality relations

$$a_p(z) = \frac{1}{N_p}\int_S (Q\vec{U}_{\perp}) \cdot \hat{\vec{U}}^*_{\tilde{p}\perp}\, ds \tag{13}$$

$$a_{\tilde{p}}(z) = \frac{1}{N_p}\int_S (Q\vec{U}_{\perp}) \cdot \hat{\vec{U}}^*_{p\perp}\, ds. \tag{14}$$

In accordance with (9) and (10), we have the following system of two excitation equations for modes p and $\tilde{p}$ (see (41) in [4])

$$\frac{da_p(z)}{dz} + \gamma_p a_p(z) = \frac{1}{N_p}\int_{S_j} \vec{F} \cdot \hat{\vec{U}}^*_{\tilde{p}\perp}\, ds \tag{15}$$

$$\frac{da_{\tilde{p}}(z)}{dz} + \gamma_{\tilde{p}} a_{\tilde{p}}(z) = \frac{1}{N_p}\int_{S_j} \vec{F} \cdot \hat{\vec{U}}^*_{p\perp}\, ds \tag{16}$$

where

$$\vec{F} = -\begin{pmatrix} \vec{j}^{\,el}_{\perp} - \frac{i}{\omega\mu}\nabla_{\perp} \times \vec{j}^{\,mag}_{||} \\ \vec{j}^{\,mag}_{\perp} + \frac{i}{\omega\epsilon}\nabla_{\perp} \times \vec{j}^{\,el}_{||} \end{pmatrix} \tag{17}$$

S_j is the cross section of the currents' region (the region of excitation).

After integration by parts with the use of the Maxwell equations, one can obtain

$$\int_{S_j} \vec{F} \cdot \hat{\vec{U}}^*_{\tilde{p}\perp}\, ds = \int_{S_j} (\vec{f}_{\perp} \cdot \hat{\vec{U}}^*_{\tilde{p}\perp} + \vec{f}_{||} \cdot \hat{\vec{U}}^*_{\tilde{p}_{||}})\, ds + \oint_{l_j} \vec{U}_{ex} \cdot (R_j \hat{\vec{U}}^*_{\tilde{p}\perp})\, dl \tag{18}$$

where l_j is the contour surrounding the cross section S_j, and $\vec{f}_{\perp,||}$ are the vector functions

$$\vec{f}_{\perp,||} = -\begin{pmatrix} \vec{j}^{\,el}_{\perp,||} \\ \vec{j}^{\,mag}_{\perp,||} \end{pmatrix}, \quad \vec{f} = \vec{f}_{\perp} + \vec{f}_{||} \tag{19}$$

R_j is the matrix

$$R_j = \begin{pmatrix} 0 & \vec{n}_j\times \\ -\vec{n}_j\times & 0 \end{pmatrix} \tag{20}$$

$\vec{n}_j$ is the unit vector along the external normal to the contour l_j. We have an analogous relation for the integral in the right-hand side of (16).

Evidently, the spectral method includes the results obtained from the reciprocity theorem and gives the contour integral as an additional term in the equation usually obtained from the reciprocity relation [1]. On the other hand, one can see that the combination of the spectral method and the reciprocity relation (see (42) in [4]) is superfluous. An important conclusion follows from our analysis. One has two possibilities to describe the mode excitation problem in waveguides if longitudinal currents take place. The first possibility is based on using delta functions caused by abrupt discontinuities of longitudinal currents [see (15) and (17)]; the second is to use the contour integral on the boundary of the currents' region [see (18)]. We will conduct our investigations in this paper by taking the contour integrals into consideration.

In the coupled-mode theory polarization currents have to be considered. For dielectric guide structures we have an electrical polarization current [1]. For more complex guide structures we have both electrical and magnetic polarization currents. One meets such a case in the analysis of mode coupling in chirowaveguides [28]. To illustrate an application of the spectral method for the coupled-mode theory, we complete this section with an example of dielectric guide structure.

Let us consider closed cylindrical waveguide filled by homogeneous dielectric medium. Without any loss of generality, one can suppose to have vacuum with permittivity ϵ_o. Normal modes of such a waveguide are classified as TE and TM modes. Let an isotropic dielectric cylindrical insert with permittivity ϵ be placed inside the waveguide (Fig. 1). Our consideration will be restricted to one insert, but it may be easily extended for a number of dielectric inserts.

An excitation of normal modes of the basic system takes place by means of the electrical polarization current [1]

$$\vec{j}^{\,el} = i\omega\Delta\epsilon\vec{E} \tag{21}$$

where

$$\Delta\epsilon = \epsilon - \epsilon_o. \tag{22}$$

The magnetic polarization current is equal to zero. By using (9) and (10), one can obtain

$$\vec{j}^{\,el}_{\perp} = i\omega\Delta\epsilon\left\{\sum_m a_m(z)\hat{\vec{E}}_{m_\perp} + \sum_{\vartheta,\tilde{\vartheta}}[a_\vartheta(z)\hat{\vec{E}}_{\vartheta_\perp} + a_{\tilde{\vartheta}}(z)\hat{\vec{E}}_{\tilde{\vartheta}_\perp}]\right\} \tag{23}$$

$$\vec{j}^{\,el}_{||} = i\omega\epsilon_o\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}\left\{\sum_m a_m(z)\hat{\vec{E}}_{m_{||}} + \sum_{\vartheta,\tilde{\vartheta}}[a_\vartheta(z)\hat{\vec{E}}_{\vartheta_{||}} + a_{\tilde{\vartheta}}(z)\hat{\vec{E}}_{\tilde{\vartheta}_{||}}]\right\}. \tag{24}$$

On the basis of (15), (16), (18), (23), and (24), we have the excitation equations for evanescent modes

$$\begin{aligned}\frac{da_p(z)}{dz} &+ \gamma_p a_p(z)\\ = &-\frac{i\omega}{N_p}\int_{S_j}\Delta\epsilon\sum_{\vartheta,\tilde{\vartheta}}\\ &\cdot\left[a_\vartheta(z)\left(\hat{\vec{E}}_{\vartheta_\perp}\cdot\hat{\vec{E}}^*_{\tilde{p}_\perp} + \frac{\epsilon_o}{\epsilon_o+\Delta\epsilon}\hat{\vec{E}}_{\vartheta_{||}}\cdot\hat{\vec{E}}^*_{\tilde{p}_{||}}\right)\right.\\ &\left.+ a_{\tilde{\vartheta}}(z)\left(\hat{\vec{E}}_{\tilde{\vartheta}_\perp}\cdot\hat{\vec{E}}^*_{\tilde{p}_\perp} + \frac{\epsilon_o}{\epsilon_o+\Delta\epsilon}\hat{\vec{E}}_{\tilde{\vartheta}_{||}}\cdot\hat{\vec{E}}^*_{\tilde{p}_{||}}\right)\right]ds\\ &-\frac{1}{N_p}\oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}\sum_{\vartheta,\tilde{\vartheta}}[a_\vartheta(z)\hat{\vec{E}}_{\vartheta_{||}} + a_{\tilde{\vartheta}}(z)\hat{\vec{E}}_{\tilde{\vartheta}_{||}}]\\ &\cdot(\vec{n_j}\times\hat{\vec{H}}^*_{\tilde{p}_\perp})\,dl\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{da_{\tilde{p}}(z)}{dz} &+ \gamma_{\tilde{p}} a_{\tilde{p}}(z)\\ = &-\frac{i\omega}{N_p}\int_{S_j}\Delta\epsilon\sum_{\vartheta,\tilde{\vartheta}}\\ &\cdot\left[a_\vartheta(z)\left(\hat{\vec{E}}_{\vartheta_\perp}\cdot\hat{\vec{E}}^*_{p_\perp} + \frac{\epsilon_o}{\epsilon_o+\Delta\epsilon}\hat{\vec{E}}_{\vartheta_{||}}\cdot\hat{\vec{E}}^*_{p_{||}}\right)\right.\\ &\left.+ a_{\tilde{\vartheta}}(z)\left(\hat{\vec{E}}_{\tilde{\vartheta}_\perp}\cdot\hat{\vec{E}}^*_{p_\perp} + \frac{\epsilon_o}{\epsilon_o+\Delta\epsilon}\hat{\vec{E}}_{\tilde{\vartheta}_{||}}\hat{\vec{E}}^*_{p_{||}}\right)\right]ds\\ &-\frac{1}{N_p}\oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}\sum_{\vartheta,\tilde{\vartheta}}[a_\vartheta(z)\hat{\vec{E}}_{\vartheta_{||}} + a_{\tilde{\vartheta}}(z)\hat{\vec{E}}_{\tilde{\vartheta}_{||}}]\\ &\cdot(\vec{n_j}\times\hat{\vec{H}}^*_{p_\perp})\,dl\end{aligned} \tag{26}$$

In these expressions, we have $\vartheta = 1, 2, \cdots, p, \cdots$ and $\tilde{\vartheta} = 1, 2, \cdots, \tilde{p}, \cdots$.

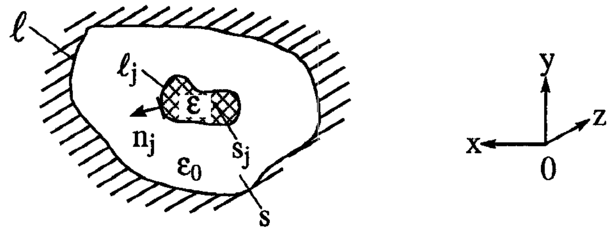

Fig. 1. A cylindrical waveguide with a dielectric insert.

One can easily obtain analogous equations for propagating modes. We will characterize the role of the contour integrals in processes of mode coupling as induced polarization effects.

III. Mode Coupling and Energetic Relations

In the coupled-mode theory, an analysis of power transfer between modes is a very useful tool [1]–[3]. Such an analysis is necessary also in our case, when induced polarization effects are taken into account. Let us write the Maxwell operator (6) in the next form [4]

$$M\vec{U} = \left(M_\perp + \frac{\partial}{\partial z}Q\right)\vec{U} \tag{27}$$

where $M_\perp$ is the operator similar to the operator M but operating only over transverse coordinates. By using such a representation for inhomogeneous Maxwell equations (5) and for the complex conjugated form of these equations, one obtains

$$\begin{aligned}&\int_{S_j}[(M_\perp\vec{U})\vec{U}^* + \vec{U}(M_\perp\vec{U})^*]\,ds\\ &\quad+\int_{S_j}\left[\left(\frac{\partial}{\partial z}Q\vec{U}\right)\vec{U}^* + \vec{U}\left(\frac{\partial}{\partial z}Q\vec{U}\right)^*\right]ds\\ &= \int_{S_j}(\vec{f}\cdot\vec{U}^* + \vec{U}\cdot\vec{f}^*)\,ds.\end{aligned} \tag{28}$$

For homogeneous boundary conditions, the first integral in the left-hand side of (28) is equal to zero. The second integral in the left-hand side of (28) corresponds to the variation of power flow along the z-axis. Thus, in such a case, the variation of power flow along the z-axis is determined by the excitation integral in the right-hand side of (28).

However, another situation exists when the longitudinal part of electromagnetic field is expressed by (10). Because of inhomogeneous boundary condition, caused by the field $\vec{U}_{ex}$, the first integral in the left-hand side of (28) will not be equal to zero. An exchange of power between interacting modes has special features in such a case.

Let us consider two propagating modes p and q. We have on the basis of (9) and (10)

$$\vec{U} = \vec{U}_\perp + \vec{U}_{||} = \vec{U}_\Sigma + \vec{U}_{ex} \tag{29}$$

where

$$\vec{U}_\Sigma = a_p(z)(\hat{\vec{U}}_{p_\perp} + \hat{\vec{U}}_{p_{||}}) + a_q(z)(\hat{\vec{U}}_{q_\perp} + \hat{\vec{U}}_{q_{||}}).$$

When we substitute (29) into (28), we can see that the second integral in the left-hand side of (28) is expressed as

$$\int_{S_j}\left[\left(\frac{\partial}{\partial z}Q\vec{U}\right)\vec{U}^* + \vec{U}\left(\frac{\partial}{\partial z}Q\vec{U}\right)^*\right]ds = N_p\frac{d(a_p a_p^*)}{dz} + N_q\frac{d(a_q a_q^*)}{dz} \tag{30}$$

by using the relations of mode orthogonality. The integral in the right-hand side of (28) is equal to zero. This is not so difficult to show when (11) and the relation (21) for the polarization current are taken into account.

Now we dwell on the first integral in the left-hand side of (28). After some transformations using the procedure of integration by parts, one can be convinced that

$$J \equiv \int_{S_j}[(M_\perp\vec{U})\cdot\vec{U}^* + \vec{U}(M_\perp\vec{U})^*]\,ds = -\oint_{l_j}[\vec{U}_\Sigma(R_j\vec{U}^*_{ex}) + \vec{U}^*_\Sigma(R_j\vec{U}^*_{ex})]\,dl \tag{31}$$

where R_j is the matrix (20). For electric polarization current with the use of (11) and (24), we have, for two propagating modes

$$J = \oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}[a_p a_q^*(\hat{\vec{E}}_{p_{||}}\times\hat{\vec{H}}^*_{q_\perp} + \hat{\vec{E}}^*_{q_{||}}\times\hat{\vec{H}}_{p_\perp}) + a_p^* a_q(\hat{\vec{E}}^*_{p_{||}}\times\hat{\vec{H}}_{q_\perp} + \hat{\vec{E}}_{q_{||}}\times\hat{\vec{H}}^*_{p_\perp})]\cdot\vec{n}_j\,dl. \tag{32}$$

For the propagating modes, all vector products in (32) are pure imaginary (one can be convinced with such an assertion on the basis of further consideration). Therefore, as a result, we can represent (28) as

$$N_p\frac{d(a_p a_p^*)}{dz} + N_q\frac{d(a_q a_q^*)}{dz} = (a_p^* a_q - a_p a_q^*)\oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}\cdot(\hat{\vec{E}}^*_{p_{||}}\times\hat{\vec{H}}_{q_\perp} - \hat{\vec{E}}^*_{q_{||}}\times\hat{\vec{H}}_{p_\perp})\cdot\vec{n}_j\,dl. \tag{33}$$

One can see that the term in the right-hand side of (33) is real.

Our analysis shows that because of an asymmetry in the field structure of two propagating modes, we have an asymmetry in the coupling of these modes. Owing to induced polarization effects, one obtains exchange of active power not only between two modes (even if the phase synchronism takes place) but with all modes (propagating and evanescent) of the spectrum. By extending the analysis of energetic relations on the infinite functional space of propagating modes (see Mathematical Appendix), instead of (33), one obtains

$$\sum_{k=1}^{\infty}N_k\frac{d(a_k a_k^*)}{dz} = 0. \tag{34}$$

An asymmetry in mode coupling due to induced polarization effects results in some changes in well-known theories of mode-coupling processes.

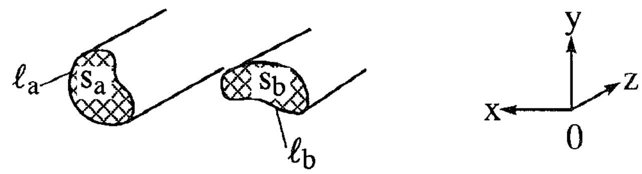

Fig. 2. A system of two parallel dielectric waveguides.

IV. Induced Polarization Effects in the Coupled-Mode Theory of Parallel Dielectric Waveguides

The general theory represented above enables us to extend the coupled-mode theory [7] for the analysis of strongly guided structures when large refractive index differences take place.

Let us consider a system of two parallel isotropic waveguides, composed of waveguides a and b and uniform in z direction (Fig. 2). Let $\epsilon^a(x,y)$ and $\epsilon^b(x,y)$ be the distributions of the permittivity of isolated waveguides a and b, correspondingly, and $\epsilon(x,y)$ be that of the composite waveguide system. We introduce the quantities

$$\Delta\epsilon^i(x,y) = \epsilon(x,y) - \epsilon^i(x,y), \qquad i = a,b \tag{35}$$

where $\Delta\epsilon^i(x,y)$ gives the perturbations in the permittivity distribution, which has a nonzero value only inside the core region.

The homogeneous Maxwell equations for the composite waveguide system can be represented as two systems of inhomogeneous equations

$$M^i\vec{U} = \vec{f}^i \tag{36}$$

where

$$M^i = \begin{pmatrix} i\omega\epsilon^i & -\nabla\times \\ \nabla\times & i\omega\mu \end{pmatrix} \tag{37}$$

$$\vec{f}^i = -i\omega\Delta\epsilon^i\begin{pmatrix}\vec{E}\\ 0\end{pmatrix}. \tag{38}$$

To solve (36), one can use the orthonormal basis of modal transverse fields. If the basis of propagating modes of the guide a is used, we have

$$\vec{U}_\perp = \vec{U}^a_\perp = \sum_{m=1}^{\infty}a_m(z)\hat{\vec{U}}^a_{m_\perp}(x,y) \tag{39}$$

and the excitation equation

$$\frac{da_m(z)}{dz} + \gamma^a_m a_m(z) = \frac{1}{N^a_m}\int_S\vec{F}^a\cdot(\hat{\vec{U}}^a_{m_\perp})^*\,ds. \tag{40}$$

When we represent the fields by the basis of propagating modes of the guide b

$$\vec{U}_\perp = \vec{U}^b_\perp = \sum_{n=1}^{\infty}b_n(z)\hat{\vec{U}}^b_{n_\perp}(x,y) \tag{41}$$

we have the excitation equation

$$\frac{db_n(z)}{dz} + \gamma^b_n b_n(z) = \frac{1}{N^b_n}\int_S\vec{F}^b\cdot(\hat{\vec{U}}^b_{n_\perp})^*\,ds. \tag{42}$$

In (40) and (42), γ^a_m, γ^b_n and N^a_m, N^b_n are, correspondingly, the propagation constants and the norms of modes. The functions $\vec{F}^a$ and $\vec{F}^b$ are defined on the basis of (17). For our case of propagating modes, γ_m^a and γ_n^b are pure imaginary quantities.

Now we express the transverse part of the total field $\vec{U}_\perp$ approximately as linear combinations of two guided modes of the waveguides a and b, respectively, [7]–[10]

$$\vec{U}_\perp \simeq A_m(z)\hat{\vec{U}}^a_{m_\perp}(x,y) + B_n(z)\hat{\vec{U}}^b_{n_\perp}(x,y). \tag{43}$$

The longitudinal components of the total fields are also linear combinations of the longitudinal components of two guided modes

$$\vec{U}_{||} \simeq A_m(z)D^a\hat{\vec{U}}^a_{m_{||}}(x,y) + B_n(z)D^b\hat{\vec{U}}^b_{n_{||}}(x,y). \tag{44}$$

The coefficients D^a and D^b are accepted to be equal to unit in [8]. In the papers [2] and [7], these coefficients are expressed as

$$D^i = \begin{pmatrix} \frac{\epsilon^i}{\epsilon} & 0 \\ 0 & 1 \end{pmatrix}, \qquad i = a, b. \tag{45}$$

One can see that the question concerning quantities of the coefficients D^a and D^b, is still debatable [9], [10], [29], [30].

According to (10) and (11), the longitudinal electric field is described as a series of propagating modes

$$\vec{E}_{||} = \sum_{\vartheta=1}^{\infty} a_\vartheta(z)\hat{\vec{E}}_{\vartheta_{||}}$$

in the regions without sources (i.e., in the regions where $\vec{j}^{\,el}_{||} = 0$ or $\Delta\epsilon^i = 0, i = a, b$). Therefore, in our model of the two modes representation, it is more correct to have two kinds of expressions for the total longitudinal fields: the first kind for the region where $\Delta\epsilon^a \neq 0$, the second for the region where $\Delta\epsilon^b \neq 0$. It means that for the region restricted by the cross section of the guide b, S_b (where $\Delta\epsilon^a \neq 0$) we have

$$\vec{U}^{(a)}_{||} \simeq A_m(z)D^a\hat{\vec{U}}^a_{m_{||}} + B_n(z)\hat{\vec{U}}^b_{n_{||}}. \tag{46}$$

For another region that is restricted by the cross section of the guide a, S_a (where $\Delta\epsilon^b \neq 0$), we obtain

$$\vec{U}^{(b)}_{||} \simeq A_m(z)\hat{\vec{U}}^a_{m_{||}} + B_n(z)D^b\hat{\vec{U}}^b_{n_{||}}. \tag{47}$$

The coefficients D^a and D^b are determined according to (45).

On the basis of the procedure expounded in Section II of the paper, taking into account the total field expressions (43), (46), and (47), one obtains for (40) and (42) after some transformations

$$\frac{da_m(z)}{dz} = -\gamma_m^a a_m(z) + K^{aa}_{mm}A_m(z) + K^{ab}_{mn}B_n(z) \tag{48}$$

$$\frac{db_n(z)}{dz} = -\gamma_n^b b_n(z) + K^{bb}_{nn}B_n(z) + K^{ba}_{nm}A_m(z) \tag{49}$$

where

$$K^{aa}_{mm} = \frac{1}{N^a_m}\Bigg\{-i\omega\int_{S_b}\Delta\epsilon^a\left[\hat{\vec{E}}^a_{m_\perp}\cdot(\hat{\vec{E}}^a_{m_\perp})^* + \frac{\epsilon^a}{\epsilon^a+\Delta\epsilon^a}\cdot\hat{\vec{E}}^a_{m_{||}}\cdot(\hat{\vec{E}}^a_{m_{||}})^*\right]ds + \oint_{lb}\frac{\Delta\epsilon^a}{\epsilon^a+\Delta\epsilon^a}\cdot[\hat{\vec{E}}^a_{m_{||}}\times(\hat{\vec{H}}^a_{m_\perp})^*]\vec{n}_b\,dl\Bigg\} \tag{50}$$

$$K^{ab}_{mn} = \frac{1}{N^a_m}\Bigg\{-i\omega\int_{Sb}\Delta\epsilon^a[\hat{\vec{E}}^b_{n_\perp}\cdot(\hat{\vec{E}}^a_{m_\perp})^* + \hat{\vec{E}}^b_{n_{||}}\cdot(\hat{\vec{E}}^a_{m_{||}})^*]\cdot ds + \oint_{lb}\frac{\Delta\epsilon^a}{\epsilon^a}[\hat{\vec{E}}^b_{n_{||}}\times(\hat{\vec{H}}^a_{m_\perp})^*]\cdot\vec{n}_b\,dl\Bigg\}. \tag{51}$$

One finds the coefficients K^{bb}_{nn} and K^{ba}_{nm} analogously by replacing the indexes a and b and the indexes m and n.

In these expressions, l_a and l_b are the contours surrounding the waveguide cross sections, correspondingly, S_a and S_b. n_a and n_b are the normals to the contours l_a and l_b (Fig. 2).

Now we will use the traditional procedure of the Galerkin method [26] to solve the approximate equation (43). On the basis of the representation (39) and using orthogonality relations, one obtains

$$a_m(z) = A_m(z) + C^{ab}_{mn}B_n(z). \tag{52}$$

Analogously, the use of (41) gives

$$b_n(z) = B_n(z) + C^{ba}_{nm}A_m(z). \tag{53}$$

The coefficients C^{ab}_{mn} and C^{ba}_{nm} describe the mode overlap

$$C^{ab}_{mn} = \frac{1}{N^a_m}\int_S(Q\hat{\vec{U}}^b_{n_\perp})\cdot(\hat{\vec{U}}^a_{m_\perp})^*\,ds \tag{54}$$

$$C^{ba}_{nm} = \frac{1}{N^b_n}\int_S(Q\hat{\vec{U}}^a_{m_\perp})\cdot(\hat{\vec{U}}^b_{n_\perp})^*\,ds. \tag{55}$$

After substituting (54) and (55) into (48) and (49), we have the coupled-mode equations

$$\frac{dA_m(z)}{dz} = \delta^a A_m(z) + k^{ab}B_n(z) \tag{56}$$

$$\frac{dB_n(z)}{dz} = \delta^b B_n(z) + k^{ba}A_m(z) \tag{57}$$

where

$$\delta^a = -\gamma^a_m + [K^{aa}_{mm} - C^{ab}_{mn}K^{ba}_{nm} + C^{ab}_{mn}C^{ba}_{nm}(\gamma^b_n - \gamma^a_m)]/(1 - C^{ab}_{mn}C^{ba}_{nm}) \tag{58}$$

$$\delta^b = -\gamma^b_n + [K^{bb}_{nn} - C^{ba}_{nm}K^{ab}_{mn} + C^{ab}_{mn}C^{ba}_{nm}(\gamma^a_m - \gamma^b_n)]/(1 - C^{ab}_{mn}C^{ba}_{nm}) \tag{59}$$

$$k^{ab} = [K^{ab}_{mn} + C^{ab}_{mn}(\gamma^b_n - \gamma^a_m - K^{bb}_{nn})]/(1 - C^{ab}_{mn}C^{ba}_{nm}) \tag{60}$$

$$k^{ba} = [K^{ba}_{nm} + C^{ba}_{nm}(\gamma^a_m - \gamma^b_n - K^{aa}_{mm})]/(1 - C^{ab}_{mn}C^{ba}_{nm}). \tag{61}$$

The expressions for the propagating constants δ^a and δ^b and the coupling coefficients k^{ab} and k^{ba} have similar forms to those obtained in [7]. Nevertheless, the coefficients $K^{aa}_{mm}, K^{bb}_{nn}, K^{ab}_{mn}$ are distinguished. The principal character of these distinctions is displayed by taking into account the contour integrals, which describe the singularities caused by abrupt discontinuities of the permittivity.

In [7], Hardy and Streifer have shown that even in the lossless case the coupling coefficients k^{ab} and k^{ba} are not complex conjugates for nonidentical waveguides. It is caused by nonidentity of $\Delta\epsilon^a$ and $\Delta\epsilon^b$. An asymmetry in mode coupling is expressed by the overlap integrals C^{ab}_{mn} and C^{ba}_{nm}. In our analysis, we have demonstrated that taking into account the induced polarization effects gives additional asymmetry in the mode coupling. This is considered when the TE-TM or TM-TM coupling takes place.

Our theory is a necessary supplement to the improved coupled-mode theory [7] for optical waveguides with large refractive index differences.

V. Role of Geometric Effects at Mode Coupling in Isotropic Discretely Inhomogeneous Waveguides

In this section, the geometric (induced polarization) effects on mode coupling in IDIW will be shown in another application of the theory.

In different problems concerning IDIW structures, one can use different types of sets of TE and TM modes: propagating or evanescent modes of a hollow waveguide. If a model of IDIW structure is used for the analysis of optical waveguides, the cross section of hollow metallic waveguide has to be large enough [21]–[23], and one has to use a set of propagating TE and TM modes. On the other hand, the frequency regions of millimeter or submillimeter IDIW correspond to near cutoff frequencies of hollow metallic waveguide with the same geometry of cross section. Moreover, complex and backward modes in IDIW usually take place in the region of cutoff frequencies of a corresponding hollow waveguide [15]–[20]. The analysis of millimeter or submillimeter IDIW has to be based, therefore, on the sets of propagating and evanescent modes.

Let us consider the IDIW structure shown in Fig. 1. For two forward propagating modes (TE mode m with the propagation constant $\gamma_m = i\beta_m$ and TM mode n with the propagation constant $\gamma_n = i\beta_n$), by using the analysis in Section II, one obtains

$$\frac{d}{dz}\begin{pmatrix} a_m \\ a_n \end{pmatrix} = \begin{pmatrix} -i\beta_m + C_{mm} & C_{mn} \\ C_{nm} & -i\beta_n + C_{nn} \end{pmatrix}\begin{pmatrix} a_m \\ a_n \end{pmatrix} \tag{62}$$

where

$$C_{mm} = -\frac{i\omega}{N_m}\int_{S_j} \Delta\epsilon \hat{\vec{E}}_{m\perp}\cdot\hat{\vec{E}}^*_{m\perp}\,ds \tag{63}$$

$$C_{nn} \equiv C'_{nn} + F_n \tag{64}$$

$$C'_{nn} = -\frac{i\omega}{N_n}\int_{S_j}\Delta\epsilon \cdot\left(\hat{\vec{E}}_{n\perp}\cdot\hat{\vec{E}}^*_{n\perp} + \frac{\epsilon_o}{\epsilon_o+\Delta\epsilon}\hat{\vec{E}}_{n\|}\cdot\hat{\vec{E}}^*_{n\|}\right)ds \tag{65}$$

$$F_n = \frac{1}{N_n}\oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}(\hat{\vec{E}}_{n\|}\times\hat{\vec{H}}^*_{n\perp})\cdot\vec{n}_j\,dl \tag{66}$$

$$C_{mn} \equiv C'_{mn} + K_{mn} \tag{67}$$

$$C'_{mn} = -\frac{i\omega}{N_m}\int_{S_j}\Delta\epsilon\hat{\vec{E}}_{n\perp}\cdot\hat{\vec{E}}^*_{m\perp}\,ds \tag{68}$$

$$K_{mn} = \frac{1}{N_m}\oint_{l_j}\frac{\Delta\epsilon}{\epsilon_o+\Delta\epsilon}(\hat{\vec{E}}_{n\|}\times\hat{\vec{H}}^*_{m\perp})\cdot\vec{n}_j\,dl \tag{69}$$

$$C_{nm} = -\frac{i\omega}{N_n}\int_{S_j}\Delta\epsilon\hat{\vec{E}}_{m\perp}\cdot\hat{\vec{E}}^*_{n\perp}\,ds. \tag{70}$$

One can see that, in comparison with expressions usually obtained for the coupling coefficients (the expressions for C_{mm}, C'_{nn}, C'_{mn}, and C_{nm}), we have additional terms caused by the contour integrals (the expressions for F_n and K_{mn}). For a wavelike solution of (62), which is determined by the factor $e^{-\gamma z}$, one obtains

$$\begin{aligned}\gamma^2 &+ [-i(\beta_m+\beta_n) + C_{mm} + C'_{nn} + F_n]\gamma \\ &+ (i\beta_m - C_{mm})(i\beta_n - C'_{nn} - F_n) + |C'_{mn}|^2 \\ &+ (C'_{mn})^* K_{mn} = 0.\end{aligned} \tag{71}$$

Here we suppose that $N_m = N_n$. In such a case, one has

$$C_{nm} = -(C'_{mn})^*. \tag{72}$$

It means that without the terms K_{mn} the matrix of coupling coefficients is skew–Hermitian. Analogous expressions can be obtained for two propagating TM modes. In a similar way, one can consider the TE-TM and TM-TM couplings of two forward and backward propagating modes.

Let modes m and $\tilde{m}$ be two conjugate TE evanescent modes and modes n and $\tilde{n}$ be two conjugate TM evanescent modes. The propagation constants of modes are $\gamma_m = \alpha_m, \gamma_{\tilde{m}} = -\alpha_m, \gamma_n = \alpha_n, \gamma_{\tilde{n}} = -\alpha_n$.

On the basis of (25) and (26), one obtains the coupled mode equations shown in (73) at the bottom of the page.

Analogous to the coupling considered above for propagating modes, some coupling coefficients in (73) also have terms with the contour integrals.

Our analysis shows that both for propagating and evanescent modes, the matrix of the coupling coefficients is asymmetrical because of induced polarization effects. For an arbitrary type of two propagating modes, one can obtain, on the basis of the energetic relations from Section III

$$\begin{aligned}N_p\frac{d(a_pa_p^*)}{dz} &+ N_q\frac{d(a_qa_q^*)}{dz} \\ &= a_pa_q^*(N_qK_{qp} + N_pK^*_{pq}) + a_p^*a_q(N_pK_{pq} + N_qK^*_{qp})\end{aligned} \tag{74}$$

where the coefficients K_{pq} and K_{qp} are determined by (69). An analysis of the field structure of propagating modes in conventional guides (for example, in hollow waveguides [31]) shows that when the relation (72) takes place, the coefficients K_{pq} and K_{qp} are purely imaginary. Therefore, we have

$$\begin{aligned}N_p\frac{d(a_pa_p^*)}{dz} &+ N_q\frac{d(a_qa_q^*)}{dz} \\ &= (a_p^*a_q - a_pa_q^*)(N_pK_{pq} + N_qK^*_{qp}).\end{aligned} \tag{75}$$

$$\frac{d}{dz}\begin{pmatrix} a_m \\ a_{\tilde{m}} \\ a_n \\ a_{\tilde{n}} \end{pmatrix} = \begin{pmatrix} -\alpha_m + C_{mm} & C_{m\tilde{m}} & C_{mn} & C_{m\tilde{n}} \\ C_{\tilde{m}m} & \alpha_m + C_{\tilde{m}\tilde{m}} & C_{\tilde{m}n} & C_{\tilde{m}\tilde{n}} \\ C_{nm} & C_{n\tilde{m}} & -\alpha_n + C_{nn} & C_{n\tilde{n}} \\ C_{\tilde{n}m} & C_{\tilde{n}\tilde{n}} & C_{\tilde{n}n} & \alpha_n + C_{\tilde{n}\tilde{n}} \end{pmatrix}\begin{pmatrix} a_m \\ a_{\tilde{m}} \\ a_n \\ a_{\tilde{n}} \end{pmatrix} \tag{73}$$

VI. Mode Coupling in a Rectangular Waveguide with a Dielectric Insert—Some Numerical Estimates

To estimate the importance of taking into account induced polarization effects in the mode coupling numerically, we adduce here some numerical examples for a rectangular waveguide with a dielectric insert. We will analyze the structure used in [16], [17]. Our results may easily be extended for another structure of IDIW.

Let us consider the coupling of two forward propagating modes: TE mode m and TM mode n. On the basis of the known expressions for the fields in a rectangular waveguide [31] and corresponding relations for the norm N_m and N_n, one can obtain the expressions for the coupling coefficients (63)–(70). With the use of the following designations

$$k_x^{m,n} = \frac{r^{m,n}\pi}{a}, \quad k_y^{m,n} = \frac{s^{m,n}\pi}{b} \tag{76}$$

$r^m, r^n, s^n, = 1, 2, \cdots . \; s^m = 0, 1, 2, \cdots$

$$k_x^{sb} = \frac{k_x^n - k_x^m}{2}, \quad k_x^{ad} = \frac{k_x^n + k_x^m}{2} \tag{77}$$

$$k_y^{sb} = k_y^n - k_y^m, \quad k_y^{ad} = k_y^n + k_y^m \tag{78}$$

we obtain

$$\begin{aligned} C_{mm} &= -i\frac{\omega\Delta\epsilon}{N_m}\int_0^h \int_{(a-t)/2}^{(a+t)/2} (E_{m_x}^2 + E_{m_y}^2)\, dx\, dy \\ &= -i\frac{\omega^2\mu_o\Delta\epsilon th}{2\beta_m ab[(k_x^m)^2 + (k_y^m)^2]}\Bigg\{(k_y^m)^2 \\ &\cdot \left(1 + \cos k_x^m a \frac{\sin k_x^m t}{k_x^m t}\right) \\ &\cdot \left(1 - \frac{\sin 2k_y^m h}{2k_y^m h}\right) + (k_x^m)^2\left(1 - \cos k_x^m a\frac{\sin k_x^m t}{k_x^m t}\right) \\ &\cdot \left(1 + \frac{\sin 2k_y^m h}{2k_y^m h}\right)\Bigg\} \end{aligned} \tag{79}$$

$$\begin{aligned} C'_{nn} &= -i\frac{\omega\Delta\epsilon}{N_n}\int_0^h \int_{(a-t)/2}^{(a+t)/2} \left(E_{n_x}^2 + E_{n_y}^2 + \frac{\epsilon_o}{\epsilon_o + \Delta\epsilon}E_{n_z}^2\right) \\ &\cdot dx\, dy \\ &= -i\Delta\epsilon\frac{th}{2ab}\Bigg\{\frac{\beta_n}{[(k_x^n)^2 + (k_y^n)^2]\epsilon_o} \\ &\cdot \Bigg[(k_x^n)^2\left(1 + \cos k_x^n a\frac{\sin k_x^n t}{k_x^n t}\right)\left(1 - \frac{\sin 2k_y^n h}{2k_y^n h}\right) \\ &+ (k_y^n)^2\left(1 - \cos k_x^n a\frac{\sin k_x^n t}{k_x^n t}\right)\left(1 + \frac{\sin 2k_y^n h}{2k_y^n h}\right)\Bigg] \\ &+ \frac{(k_x^n)^2 + (k_y^n)^2}{\beta_n(\epsilon_o + \Delta\epsilon)}\left(1 - \cos k_x^n a\frac{\sin k_x^n t}{k_x^n t}\right) \\ &\cdot \left(1 - \frac{\sin 2k_y^n h}{2k_y^n h}\right)\Bigg\} \end{aligned} \tag{80}$$

$$\begin{aligned} C'_{mn} &= -i\frac{\omega\Delta\epsilon}{N_m}\int_0^h \int_{(a-t)/2}^{(a+t)/2} (E_{m_x}E_{n_x}^* + E_{m_y}E_{n_y}^*)\, dx\, dy \\ &= -i\frac{\omega\Delta\epsilon th}{2\sqrt{[(k_x^m)^2 + (k_y^m)^2][(k_x^n)^2 + (k_y^n)^2]}ab} \\ &\cdot \sqrt{\frac{\mu_o\beta_n}{\epsilon_o\beta_m}}\Bigg[k_x^n k_y^m\left(\cos k_x^{sb}a\frac{\sin k_x^{sb}t}{k_x^{sb}t} + \cos k_x^{ad}a\right. \\ &\left.\cdot \frac{\sin k_x^{ad}t}{k_x^{ad}t}\right)\left(\frac{\sin k_y^{sb}h}{k_y^{sb}h} - \frac{\sin k_y^{ad}h}{k_y^{ad}h}\right) \\ &- k_x^m k_y^n\left(\cos k_x^{sb}a\frac{\sin k_x^{sb}t}{k_x^{sb}t} - \cos k_x^{ad}a\frac{\sin k_x^{ad}t}{k_x^{ad}t}\right) \\ &\cdot \left(\frac{\sin k_y^{sb}h}{k_y^{sb}h} + \frac{\sin k_y^{ad}h}{k_y^{ad}h}\right)\Bigg] \end{aligned} \tag{81}$$

$$\begin{aligned} F_n &= -\frac{\Delta\epsilon}{(\epsilon_o + \Delta\epsilon)N_n}\Bigg[\left(\int_{(a-t)/2}^{(a+t)/2} E_{n_z}H_{n_x}^*\, dy\right)\Bigg|_{y=h} \\ &+ \left(\int_0^h E_{n_z}H_{n_y}^*\, dy\right)\Bigg|_{x=(a-t)/2} \\ &- \left(\int_0^h E_{n_z}H_{n_y}^*\, dy\right)\Bigg|_{x=(a+t)/2}\Bigg] \\ &= i\frac{\Delta\epsilon th}{(\epsilon_o + \Delta\epsilon)\beta_n ab}\Bigg[(k_y^n)^2\frac{\sin 2k_y^n h}{2k_y^n h} \\ &\cdot \left(1 - \cos k_x^n a\frac{\sin k_x^n t}{k_x^n t}\right) + (k_x^n)^2\cos k_x^n a\frac{\sin k_x^n t}{k_x^n t} \\ &\cdot \left(1 - \frac{\sin 2k_y^n h}{2k_y^n h}\right)\Bigg] \end{aligned} \tag{82}$$

$$\begin{aligned} K_{mn} &= -\frac{\Delta\epsilon}{(\epsilon_o + \Delta\epsilon)N_m}\Bigg[\left(\int_{(a-t)/2}^{(a+t)/2} E_{n_z}H_{m_x}^*\, dx\right)\Bigg|_{y=h} \\ &+ \left(\int_0^h E_{n_z}H_{m_y}^*\, dy\right)\Bigg|_{x=(a-t)/2} \\ &- \left(\int_0^h E_{n_z}H_{m_y}^*\, dy\right)\Bigg|_{x=(a+t)/2}\Bigg] \\ &= i\frac{\Delta\epsilon\sqrt{(k_x^n)^2 + (k_y^n)^2}}{(\epsilon_o + \Delta\epsilon)\omega ab\sqrt{(k_x^m)^2 + (k_y^m)^2}}\sqrt{\frac{\epsilon_o\beta_m}{\mu_o\beta_n}} \\ &\cdot \Bigg[k_x^m t \cdot \sin k_y^n h \cdot \cos k_y^m h\left(\cos k_x^{sb}a\frac{\sin k_x^{sb}t}{k_x^{sb}t}\right. \\ &\left.- \cos k_x^{ad}a\frac{\sin k_x^{ad}t}{k_x^{ad}t}\right) - k_y^m h(\cos k_x^{sb}a \sin k_x^{sb}t \\ &+ \cos k_x^{ad}a \sin k_x^{ad}t)\left(\frac{\sin_y^{sb} h}{k_y^{sb}h} - \frac{\sin k_y^{ad}h}{k_y^{ad}h}\right)\Bigg]. \end{aligned} \tag{83}$$

In (81) and (83), we have used the condition $N_m = N_n$.

We have obtained (79)–(83) for $k_x^{m,n} \neq 0, k_y^{m,n} \neq 0$. For $k_y^m = 0$, one obtains

$$C_{mm} = -i\frac{\omega^2\mu_o\Delta\epsilon th}{2\beta_m ab}\left(1 - \cos k_x^m a\frac{\sin k_x^m t}{k_x^m t}\right). \tag{84}$$

One can be convinced that if we substitute $k_y^m = 0$ into (81) and (83), we have to divide the resulting expressions by two.

For numerical estimates, we will analyze a waveguide structure composed of a rectangular $a \times b$ waveguide and a rectangular $t \times h$ dielectric insert. We will consider K_u-band waveguide ($a = 15.799$ mm, $b = 7.899$ mm) and will analyze

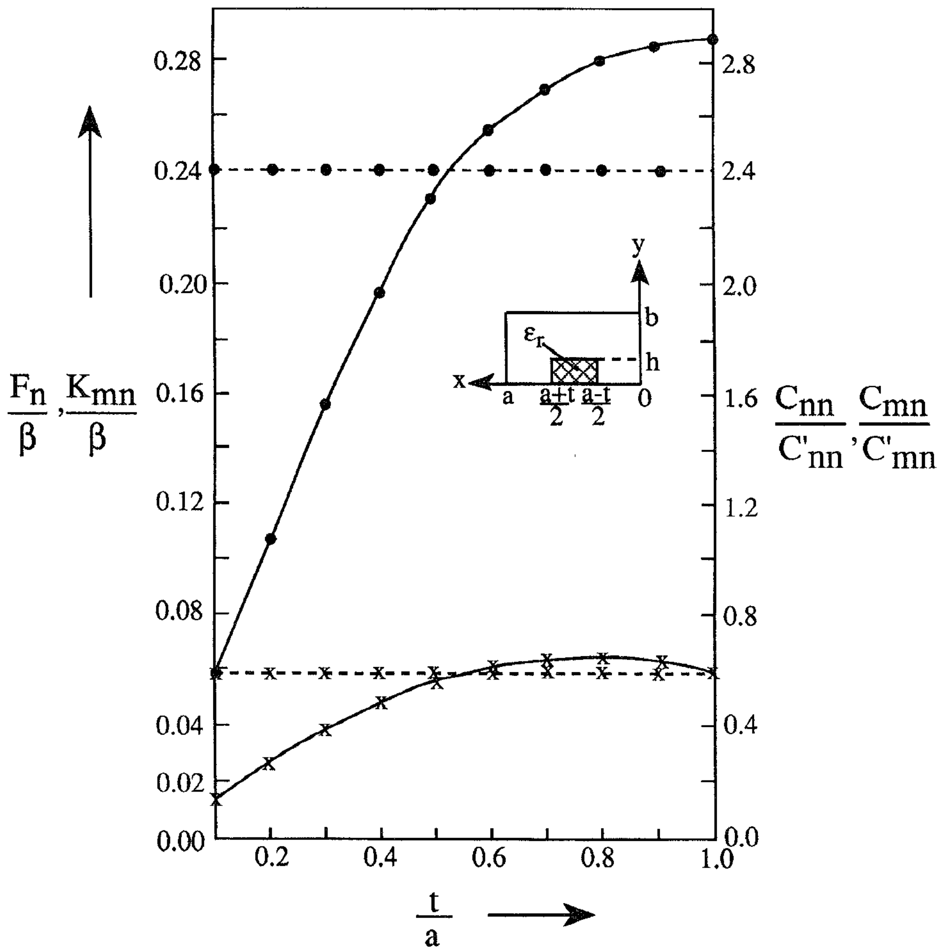

Fig. 3. Normalized coefficients F_n/β (–●–●–●–), K_{mn}/β(–×–×–×–) and normalized quantities C_{nn}/C'_{nn} (-●- -●- -●-) C_{mn}/C'_{mn} (-×- -×- -×-) as functions of normalized width t/a of a dielectric insert; $h = 0.2b$, $f = 1.1fc$, $\Delta\epsilon/\epsilon_o = 1$.

the TE-TM mode coupling for two degenerated modes. These are the modes with phase synchronism. To demonstrate the role of the induced polarization effects, we calculate the coupling coefficients for two propagating modes: TE_{11} and TM_{11}. In Fig. 3, one can see normalized coefficients $F_n/\beta, K_{mn}/\beta(\beta = \beta_m = \beta_n)$ and normalized quantities $C_{nn}/C'_{nn} = 1 + F_n/C'_{nn}$ and $C_{mn}/C'_{mn} = 1 + K_{mn}/C'_{mn}$ as a function of normalized width t/a of a dielectric insert when the height of an insert is $h = 0.2b$. Analogous dependences as a function of normalized height h/b for $t = 0.2a$ are shown in Fig. 4. The calculations were made for the normalized differences of a permittivity $\Delta\epsilon/\epsilon_o = 1$ and for frequency $f = 1, 1f_c$, where f_c is the cutoff frequency of a hollow waveguide. It is evident that induced polarization effects have considerable influence on values of coupling coefficients. One can also note a weak dependence of C_{nn}/C'_{nn} on normalized width t/a and a weak dependence of C_{mn}/C'_{mn} on both normalized width t/a and normalized height h/b.

The relations between coupling coefficients have analogous characters for below cutoff frequencies and do not adduce here.

Figs. 5–7 demonstrate how induced polarization effects influence the values of propagation constants for $TE_{11} - TM_{11}$ modes coupling. The coupling of two propagating modes may be correctly used for the analysis of dispersion characteristics of IDIW if frequencies are far from cutoff frequency. For near cutoff frequencies, one has to use the full mode coupling analysis. The curves in Fig. 7, therefore, may be less correct in comparison with the curves in Figs. 5 and 6. Nevertheless, one can see an interesting tendency concerning an appearance of backward waves caused by induced polarization effects (see Fig. 7).

VII. Conclusion

An analysis of waveguide problems based on a solution of full vector-wave equations is very important for many applications. To solve such problems, a new coupled-mode method that takes into account induced polarization effects has been proposed in this paper. This method may be a very powerful tool for the analysis of mode coupling (both propagating and evanescent modes) due to both the material and the geometric effects. One can see that, in our method we solve the first-order Maxwell equations for a full electromagnetic field, instead of the second-order vector equations used separately for electric and magnetic fields in [23] and [32].

Our method has arisen from the analysis of mathematical incorrection that takes place when one uses the reciprocity relations to solve an excitation problem of waveguides by longitudinal currents [1]. The spectral method is used to avoid

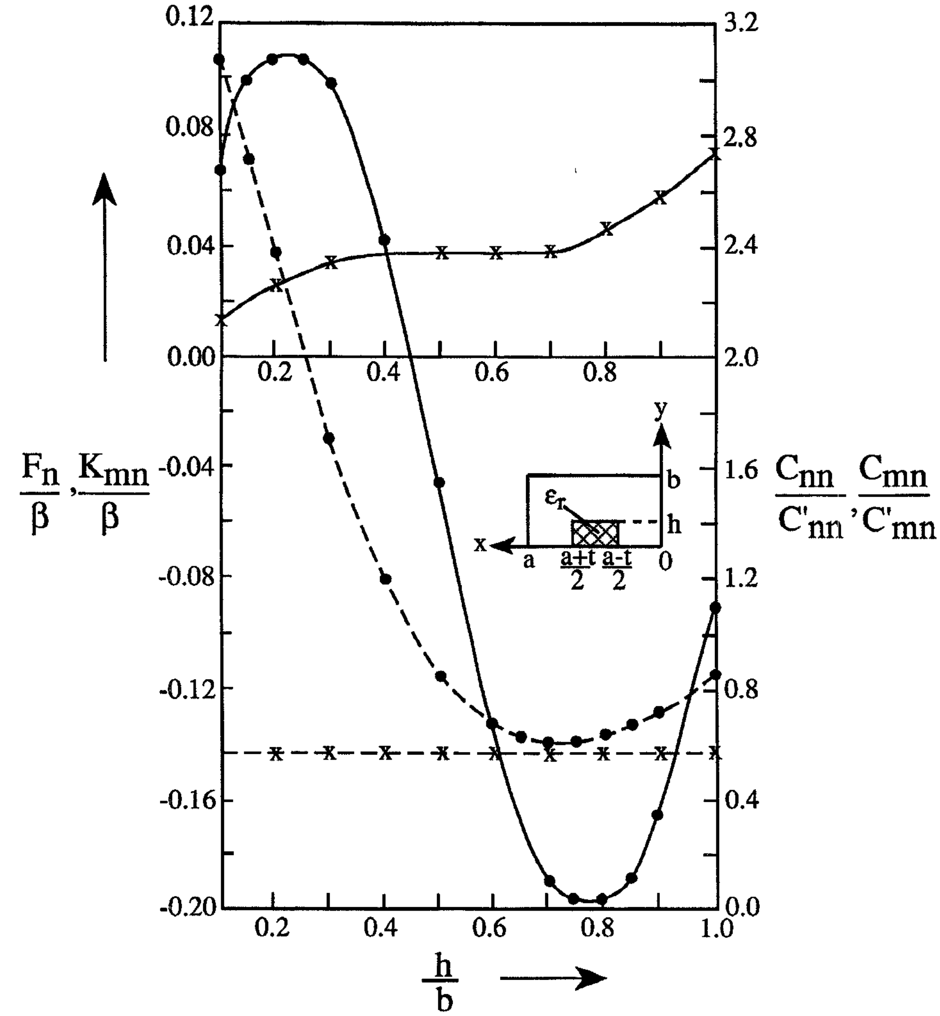

Fig. 4. Normalized coefficients F_n/β (-●-●-●-), K_{mn}/β(-×-×-×-) and normalized quantities C_{nn}/C'_{nn} (-●- -●- -●-) C_{mn}/C'_{mn} (-×- -×- -×-) as functions of normalized height h/b of a dielectric insert; $t = 0.2a,\ f = 1.1fc,\ \Delta\epsilon/\epsilon_o = 1$.

such an incorrection [4], [5]. This paper represents further development of the spectral method and shows the importance of taking into account singularities, caused by abrupt discontinuities, in mode-excitation and mode-coupling problems. One can see that such singularities may play a very important role in both open-optical and closed-microwave waveguides. Particularly, these geometric effects may be useful in the explanation of the reasons that cause an appearance of complex and backward waves. An asymmetry in mode coupling, caused by induced polarization effects, gives a non–Hermitian matrix of coupling coefficients for a finite number of coupled modes. Nevertheless, this matrix is Hermitian for an infinite number of coupled modes. Because of induced polarization effects, full exchange of active power does not take place even for phase synchronism of two propagating modes. In such a case, one obtains exchange of active power not only between two modes but with all modes (propagating and evanescent) of the spectrum.

In this paper, we have not adduced the full numerical analysis of certain waveguide structures to corroborate our theory. This may be the subject of further publications. The corroboration of the theory is based on the analysis of some waveguide problems and some numerical estimates and is, evidently, sufficiently convincing.

Mathematical Appendix

A. On the Role of Singularities in Processes of Waveguide Mode Interactions

The coupled-mode theory represents the self-matched problem with polarization currents dependent on the electromagnetic fields. In such a case, abrupt discontinuities of longitudinal polarization currents cause singularities. We have seen, for an example of two propagating modes (see Section III), that these singularities give an asymmetry in the mode coupling. In this Appendix, we will extend the analysis for an infinite number of modes and discuss the problems of convergence.

For propagating modes, we have

$$\vec{U} = \vec{U}_\Sigma + \vec{U}_{ex} \tag{A1}$$

where

$$\vec{U}_\Sigma = \vec{U}_{\Sigma_\perp} + \vec{U}_{\Sigma_{||}} = \sum_{m=1}^{\infty} a_m(z)\hat{\vec{U}}_{m_\perp} + \sum_{m=1}^{\infty} a_m(z)\hat{\vec{U}}_{m_{||}}. \tag{A2}$$

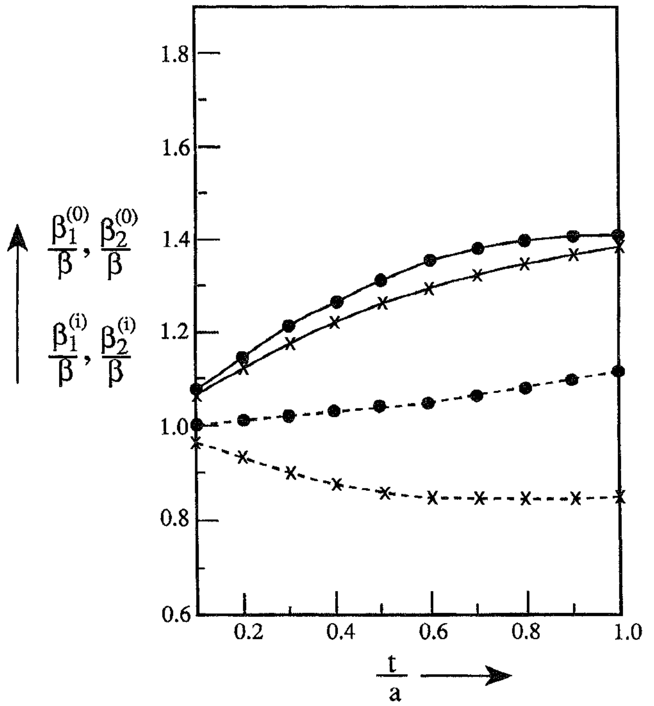

Fig. 5. Normalized propagation constants (β_1/β, β_2/β) obtained from (80) as a function of normalized width t/a. a) (-•-•-•-) $\beta_1^{(0)}/\beta$, (-x-x-x-) $\beta_1^{(i)}/\beta$ normalized propagation constant of the first wave with and without taking into account induced polarization effects. b) (-•- -•- -•-) $\beta_2^{(0)}/\beta$, (-x- -x- -x-) $\beta_2^{(i)}/\beta$ normalized propagation constant of the second wave with and without taking into account induced polarization effects. $h = 0.2b$. $f = 1.1fc$, $\Delta\epsilon/\epsilon_o = 1$.

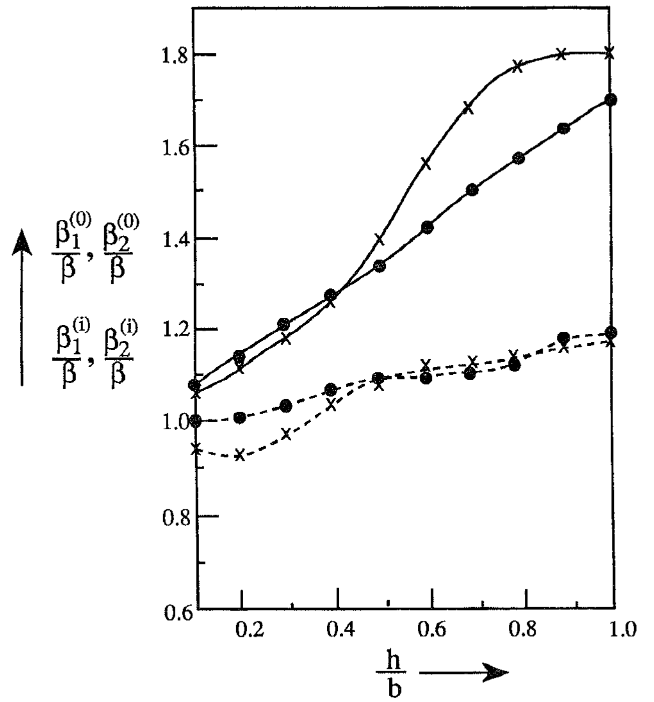

Fig. 6. Normalized propagation constants (β_1/β, β_2/β) obtained from (80) as a function of normalized height h/b. The curves are designed in the same way as in Fig 5. $t = 0.2a$, $f = 1.1fc$, $\Delta\epsilon/\epsilon_o = 1$.

Similar to the analysis conducted in Section III, one obtains

$$\sum_{m=1}^{\infty} N_m \frac{d|a_m|^2}{dz} = J \tag{A3}$$

where

$$J = -\oint_{l_j} [\vec{U}_\Sigma(R_j\vec{U}_{ex}^*) + \vec{U}_\Sigma^*(R_j\vec{U}_{ex})]\, dl. \tag{A4}$$

On the basis of (14) and (24), we have, after some transformations

$$J = -\oint_{l_j} \frac{\Delta\epsilon}{\epsilon_o + \Delta\epsilon}\left(\sum_{m=1}^{\infty} a_m \hat{\vec{E}}_{m_{||}} \times \sum_{m=1}^{\infty} a_m^* \hat{\vec{H}}^*_{m_\perp} + \sum_{m=1}^{\infty} a_m^* \hat{\vec{E}}^*_{m_{||}} \times \sum_{m=1}^{\infty} a_m \hat{\vec{H}}_{m_\perp}\right) \cdot \vec{n}_j\, dl. \tag{A5}$$

Since $\vec{E}_{m_{||}} = -(i/\omega\epsilon_o)\nabla_\perp \times \vec{H}_{m_\perp}$, one obtains

$$J = \frac{i}{\omega\epsilon_o}\oint_{l_j} \frac{\Delta\epsilon}{(\epsilon_o + \Delta\epsilon)}[(\nabla_\perp \times \vec{H}_\perp) \times \vec{H}_\perp^* - (\nabla_\perp \times \vec{H}_\perp^*) \times \vec{H}_\perp] \cdot \vec{n}_j\, dl \tag{A6}$$

where

$$\vec{H}_\perp = \sum_{m=1}^{\infty} a_m(z)\hat{\vec{H}}_{m_\perp}.$$

Because $\hat{\vec{H}}_{m_\perp}$ is a membrane function, the fields $\vec{H}_\perp$ and $\nabla_\perp \times \vec{H}_\perp$ have coinciding phases. The integral (A6) is, therefore, equal to zero and results in

$$\sum_{m=1}^{\infty} N_m \frac{d|a_m|^2}{dz} = 0. \tag{A7}$$

Let

$$\vec{a} = \begin{pmatrix} a_1(z) \\ a_2(z) \\ \vdots \\ a_m(z) \\ \vdots \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} N_1 \frac{da_1(z)}{dz} \\ N_2 \frac{da_2(z)}{dz} \\ \vdots \\ N_m \frac{da_m(z)}{dz} \\ \vdots \end{pmatrix} \tag{A8}$$

be the vectors in the finite measure space. These vectors are correlated together by the infinite matrix C

$$\vec{b} = C\vec{a}. \tag{A9}$$

Because of (A7), we have

$$\vec{a} \cdot \vec{b}^* + \vec{b} \cdot \vec{a}^* = 0 \tag{A10}$$

$$(C\vec{a})\vec{a}^* = -\vec{a}(C\vec{a})^*. \tag{A11}$$

According to the analysis conducted in the paper

$$C_{ij} \neq -C_{ji}^*. \tag{A12}$$

The matrix C is, therefore, a non-skew–Hermitian matrix. Nevertheless, the expression (A11) gives the condition of self-conjugation of the infinite matrix C. The infinite matrix C is

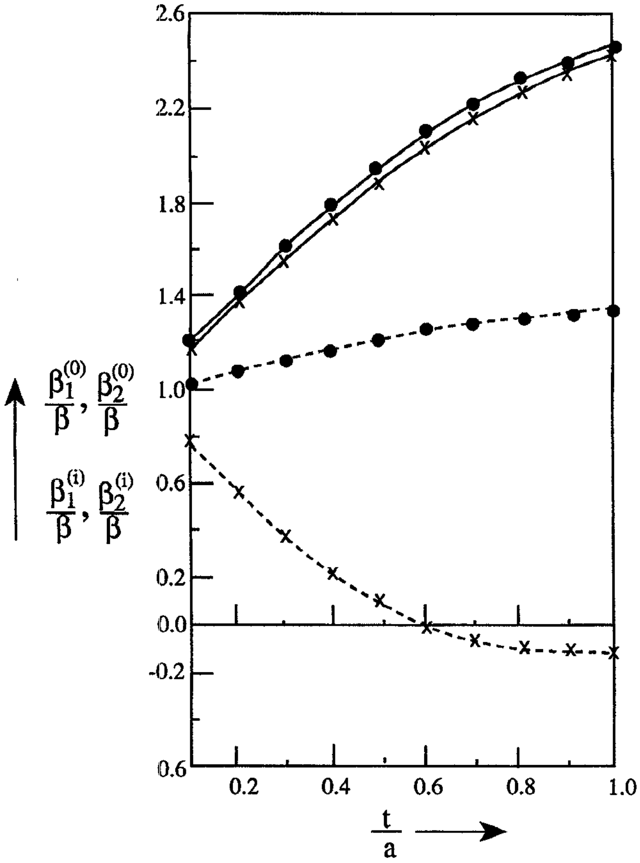

Fig. 7. Normalized propagation constants (β_1/β, β_2/β) obtained from (80) as a function of normalized width t/a for near cutoff frequency. $h = 0.2b$, $f = 1.02fc$, $\Delta\epsilon/\epsilon_o = 1$. The curves are designed in the same way as in Fig. 5.

a skew–Hermitian [33]. We can rewrite (A11) as

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}(C_{ij}a_j)a_i^* = -\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_i(C_{ij}a_j)^*, \qquad i \neq j. \tag{A13}$$

For a wavelike solution of (A9), which is determined by the factor $e^{-\gamma z}$, one obtains

$$\gamma\vec{d} = \vec{a} \tag{A14}$$

where

$$\vec{d} = \begin{pmatrix} N_1 a_1 \\ N_2 a_2 \\ \vdots \\ N_m a_m \\ \vdots \end{pmatrix}. \tag{A15}$$

Because of (A10), we have

$$\gamma^* + \gamma = 0 \tag{A16}$$

therefore, $\gamma = i\beta$.

One can make the following conclusion from our consideration. For a finite number of propagating modes, we have an exchange of active power not only between these propagating modes but also with all continuum of evanescent modes. The relation (A7) is the condition of converge for the series composed by power flows of propagating modes. Because of an asymmetry in mode coupling, the matrix of coupling coefficients is non–Hermitian. Nevertheless, the infinite matrix of coupling coefficients is self-conjugated [see (A11)]. An asymmetry in the coupling of propagating modes, caused by singularities, does not give complex modes in the spectrum. One can see that this spectrum contains only propagating modes [see (A16)].

One can carry out the analysis of interaction pairs of evanescent modes in a similar way. In this case, a finite number of pairs of evanescent modes will be exchanged by power also with all continuum of propagating modes.

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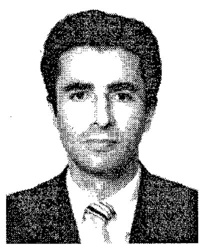

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